NAG Toolbox for MATLAB

f08he

1 Purpose

f08he reduces a real symmetric band matrix to tridiagonal form.

2 Syntax

```
[ab, d, e, q, info] = f08he(vect, uplo, kd, ab, q, 'n', n)
```

3 Description

f08he reduces a symmetric band matrix A to symmetric tridiagonal form T by an orthogonal similarity transformation:

$$T = Q^{\mathrm{T}}AQ.$$

The orthogonal matrix Q is determined as a product of Givens rotation matrices, and may be formed explicitly by the function if required.

The function uses a vectorizable form of the reduction, due to Kaufman 1984.

4 References

Kaufman L 1984 Banded eigenvalue solvers on vector machines *ACM Trans. Math. Software* **10** 73–86

Parlett B N 1998 *The Symmetric Eigenvalue Problem* SIAM, Philadelphia

5 Parameters

5.1 Compulsory Input Parameters

1: **vect – string**

Indicates whether Q is to be returned.

vect = 'V'

Q is returned.

vect = 'U'

Q is updated (and the array **q** must contain a matrix on entry).

vect = 'N'

Q is not required.

Constraint: **vect** = 'V', 'U' or 'N'.

2: **uplo – string**

Indicates whether the upper or lower triangular part of A is stored.

uplo = 'U'

The upper triangular part of A is stored.

uplo = 'L'

The lower triangular part of A is stored.

Constraint: **uplo** = 'U' or 'L'.

3: **kd – int32 scalar**

If **uplo** = 'U', the number of superdiagonals, k_d, of the matrix A.

If **uplo** = 'L', the number of subdiagonals, k_d, of the matrix A.

Constraint: **kd** ≥ 0.

4: **ab**(**ldab**,$*$) **– double array**

The first dimension of the array **ab** must be at least $\max(1, \mathbf{kd} + 1)$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

The upper or lower triangle of the n by n symmetric band matrix A.

The matrix is stored in rows 1 to $k_d + 1$, more precisely,

if **uplo** = 'U', the elements of the upper triangle of A within the band must be stored with element A_{ij} in $\mathbf{ab}(k_d + 1 + i - j, j)$ for $\max(1j - k_d) \leq i \leq j$;
if **uplo** = 'L', the elements of the lower triangle of A within the band must be stored with element A_{ij} in $\mathbf{ab}(1 + i - j, j)$ for $j \leq i \leq \min(nj + k_d)$.

5: **q**(**ldq**,$*$) **– double array**

The first dimension, **ldq**, of the array **q** must satisfy

if **vect** = 'V' or 'U', **ldq** $\geq \max(1, \mathbf{n})$;
if **vect** = 'N', **ldq** ≥ 1.

The second dimension of the array must be at least $\max(1, \mathbf{n})$ if **vect** = 'V' or 'U' and at least 1 if **vect** = 'N'

If **vect** = 'U', **q** must contain the matrix formed in a previous stage of the reduction (for example, the reduction of a banded symmetric-definite generalized eigenproblem); otherwise **q** need not be set.

5.2 Optional Input Parameters

1: **n – int32 scalar**

Default: The first dimension of the array **ab** and the second dimension of the array **ab**. (An error is raised if these dimensions are not equal.)

n, the order of the matrix A.

Constraint: **n** ≥ 0.

5.3 Input Parameters Omitted from the MATLAB Interface

ldab, ldq, work

5.4 Output Parameters

1: **ab**(**ldab**,$*$) **– double array**

The first dimension of the array **ab** must be at least $\max(1, \mathbf{kd} + 1)$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

ab contains values generated during the reduction to tridiagonal form.

The first superdiagonal and the diagonal of the tridiagonal matrix T are returned in **ab** using the same storage format as described above.

2: **d**$(*)$ **– double array**

Note: the dimension of the array **d** must be at least $\max(1, \mathbf{n})$.

The diagonal elements of the tridiagonal matrix T.

3: **e**$(*)$ **– double array**

Note: the dimension of the array **e** must be at least $\max(1, \mathbf{n} - 1)$.

The off-diagonal elements of the tridiagonal matrix T.

4: **q**(**ldq**,$*$) **– double array**

The first dimension, **ldq**, of the array **q** must satisfy

if **vect** = 'V' or 'U', **ldq** $\geq \max(1, \mathbf{n})$;
if **vect** = 'N', **ldq** ≥ 1.

The second dimension of the array must be at least $\max(1, \mathbf{n})$ if **vect** = 'V' or 'U' and at least 1 if **vect** = 'N'

If **vect** = 'V' or 'U', the n by n matrix Q.

If **vect** = 'N', **q** is not referenced.

5: **info – int32 scalar**

info = 0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

info $= -i$

If **info** $= -i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **vect**, 2: **uplo**, 3: **n**, 4: **kd**, 5: **ab**, 6: **ldab**, 7: **d**, 8: **e**, 9: **q**, 10: **ldq**, 11: **work**, 12: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

7 Accuracy

The computed tridiagonal matrix T is exactly similar to a nearby matrix $(A + E)$, where

$$\|E\|_2 \leq c(n)\epsilon\|A\|_2,$$

$c(n)$ is a modestly increasing function of n, and ϵ is the ***machine precision***.

The elements of T themselves may be sensitive to small perturbations in A or to rounding errors in the computation, but this does not affect the stability of the eigenvalues and eigenvectors.

The computed matrix Q differs from an exactly orthogonal matrix by a matrix E such that

$$\|E\|_2 = O(\epsilon),$$

where ϵ is the ***machine precision***.

8 Further Comments

The total number of floating-point operations is approximately $6n^2k$ if **vect** = 'N' with $3n^3(k-1)/k$ additional operations if **vect** = 'V'.

The complex analogue of this function is f08hs.

9 Example

```
vect = 'V';
uplo = 'L';
kd = int32(2);
ab = [4.99, 1.05, -2.31, -0.43;
     0.04, -0.79, -1.3, 0;
     0.22, 1.04, 0, 0];
q = zeros(4, 4);
[abOut, d, e, qOut, info] = f08he(vect, uplo, kd, ab, q)
```

```
abOut =
    4.9900   -2.4806   -0.0661    0.8567
    0.2236    1.1030    1.4301         0
    0.2200   -1.0930         0         0
d =
    4.9900
   -2.4806
   -0.0661
    0.8567
e =
    0.2236
    1.1030
    1.4301
qOut =
    1.0000         0         0         0
         0    0.1789   -0.1321   -0.9750
         0    0.9839    0.0240    0.1773
         0         0   -0.9909    0.1343
info =
           0
```